CERN-PH-TH/2004-118
hep-th/0406242

On-shell Gauge Invariants and Field Strengths in Open Superstring Field Theory

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June, 2004

Abstract

We study gauge invariant quantities in the open superstring field theory proposed by Berkovits, extending the precedent discussion in bosonic string field theory. Our gauge invariants are "on-shell". As its applications, we define quantities which are expected to be related to the U(1) field strength — a RR coupling and a "component" of the string field equation of motion, and consider their naive extensions to off-shell. Order by order calculations show that the field strength extracted from the RR coupling is not gauge invariant, while from the component of the equation of motion we obtain an off-shell field strength which is gauge invariant under full gauge transformation if on-shell, and under linearized gauge transformation even off-shell.

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1 Introduction

Open string field theory has been developed in the last several years, mainly in the study of tachyon condensation. However it is not yet a tool as useful as first quantized formalism when we consider other phenomena. Further development is necessary to make it as useful as first quantized formalism. Especially, in this theory it is very difficult to extract physical quantities, because of its computational complexity and nonlocality. It is helpful to know how to construct physically meaningful quantities.

String field theory is a kind of gauge theory, and gauge invariant quantities in gauge theories have some physical meanings. Therefore we want to construct gauge invariant quantities in string field theory. Studies along this direction in bosonic string field theory have been advanced in [1, 3, 2]. These works have shown that, roughly speaking, couplings of one closed string mode and one open string field are gauge invariant. In this paper we study these gauge invariants in open superstring field theory proposed by Berkovits [4]. This gauge invariant is "on-shell" because of the on-shell property of the closed string mode.

The simplest gauge invariant quantity in U(1) gauge theory is the field strength of the gauge field. Since in lowest order approximation U(1) open string field theory is equivalent to U(1) gauge theory, it is natural to consider string field theory analog of the field strength. The success of the analysis of tachyon condensation [5] in boundary string field theory (BSFT)[6] also suggests that we can define such an analog, because gauge field in BSFT transforms in the same way as ordinary gauge field, and the BSFT action is expected to be related to cubic string field theory and its superstring extension by some field redefinition.

As applications of our gauge invariant quantities, we make two attempts to extract gauge invariant field strength. Firstly, we consider string field theory counterpart of the coupling of Ramond-Ramond (p-1)-form and one field strength of Dp-brane in the effective action. From it we extract quantities analogous to gauge invariant field strength and gauge field, and extend it to off-shell in the most naive and straightforward manner. It is not clear from the general expression that the analog of gauge field transforms in the same way as ordinary one. To make it clear and to investigate how far this quantity can be extended to "off-shell", we give the component expression of this quantity up to level 2. We find that the field strength is not gauge invariant at level 2, even on-shell, and the gauge field does not transform in the same way as ordinary gauge field.

Secondly, we extract a "gauge invariant component" of the string field equation of motion.

The string field equation of motion contains a gauge invariant extension of that of ordinary gauge field i.e. $\partial_\nu F^{\mu\nu} = 0$. This string field theory counterpart has correction terms from massive fields. We compute those terms up to level 1. From this "gauge invariant component" we can extract an analog of field strength, and at the linearized level this component is gauge invariant even off-shell.

For definiteness, in this paper we consider one single D9-brane in type IIB theory. Extension of our discussion to lower dimensional D-branes is straightforward, but is restricted to one single D-brane. In section 2, we introduce couplings of one closed string mode and one open string field which are gauge invariant in Berkovits' open string field theory. Our discussion is entirely in terms of conformal field theory. In section 3, we consider coupling of RR 8-form and one open string field, and give explicit expression up to level 2. We show that field strength defined from it is not gauge invariant even on-shell. In section 4, we define another type of "on-shell" gauge invariant which reduces to the previous ones, and define gauge invariant components of the equation of motion. From one of them we extract field strength and gauge field, and show that the field strength is gauge invariant under linearized gauge transformation even off-shell. Section 5 contains discussions. In the Appendix bases for expanding string fields are tabulated.

2 "On-shell" gauge invariants

In this section we introduce gauge invariant quantities linear in string field Φ, extending the argument in [1, 3, 2] for bosonic open string field theory. Our argument is based on the conformal field theory viewpoint. If necessary, we can rewrite the following argument in terms of oscillator expression as in [2], in the case of a flat background.

The action of the NS part of the open string field theory proposed by Berkovits [4] is

$$S = \frac{1}{2g^2}\left\langle\!\!\left\langle (e^{-\Phi}Q_Be^{\Phi})(e^{-\Phi}\eta_0 e^{\Phi}) - \int_0^1 dt (e^{-t\Phi}\partial_t e^{t\Phi})\{(e^{-t\Phi}Q_Be^{t\Phi}),(e^{-t\Phi}\eta_0 e^{t\Phi})\}\right\rangle\!\!\right\rangle, \qquad (1)$$

where the string field Φ is Grassmann even, GSO(+), ghost number 0 and picture number 0 operator. CFT correlators $\langle \dots \rangle$ are defined in large Hilbert space. For details of the definition see, for instance, [7].

This action is invariant under the following gauge transformation:

$$\delta e^{\Phi} = (Q_B\Omega)e^{\Phi} + e^{\Phi}(\eta_0\Omega'), \qquad (2)$$

where the gauge transformation parameters Ω and Ω' are Grassmann odd, GSO(+) operators. Their ghost number and picture number are $(-1,0)$ and $(-1,1)$ respectively. From this expression, we can compute the transformation of Φ order by order:

$$\begin{aligned}\delta\Phi &= (Q_B\Omega) + (\eta_0\Omega') \\ &+\frac{1}{2}[(Q_B\Omega),\Phi] + \frac{1}{2}[\Phi,(\eta_0\Omega')] \\ &+\frac{1}{12}(Q_B\Omega)\Phi^2 - \frac{1}{6}\Phi(Q_B\Omega)\Phi + \frac{1}{12}\Phi^2(Q_B\Omega) \\ &+\frac{1}{12}\Phi^2(\eta_0\Omega') - \frac{1}{6}\Phi(\eta_0\Omega')\Phi + \frac{1}{12}(\eta_0\Omega')\Phi^2 \\ &+\ldots. \end{aligned} \tag{3}$$

This transformation contains infinitely many terms with arbitrarily high power in Φ.

The gauge invariant quantity $(V;\Phi)$ we consider in this paper is defined as:

$$(V;\Phi) \equiv \left\langle V(0,0)\cdot f_1^{(1)}\circ\Phi(0)\right\rangle_{\text{disk}}, \tag{4}$$

where $\langle\cdots\rangle_{\text{disk}}$ is the CFT correlation function evaluated on a unit disk with appropriate boundary condition on the boundary. Conformal mappings $f_k^{(n)}(z)$, which are used to define star products, are

$$f_k^{(n)}(z) = e^{2\pi i(k-1)/n}\left(\frac{1+iz}{1-iz}\right)^{2/n}. \tag{5}$$

$V(z,\bar z)$ is a closed string vertex operator that satisfies the following conditions:

- $[Q_B, V(z,\bar z)] = 0$, i.e. $V(z,\bar z)$ is BRST invariant.
- $[\eta_0, V(z,\bar z)] = 0$, i.e. $V(z,\bar z)$ is in small Hilbert space.
- $V(z,\bar z)$ is a dimension $(0,0)$ primary field.

Ordinary on-shell closed string vertex operators in first quantized formalism satisfy all of these conditions.

There is a subtlety in the definition of $(V;\Phi)$. The mapping $f_1^{(1)}$ is singular at the center of the unit disk where V is inserted. Geometrically, this mapping glues the left half and the right half of the open string and therefore the midpoint is singular. However, in (4), $f_1^{(1)}$ acts only on Φ at the point where this mapping is not singular. Hence we can forget that the unit disk is formed by the gluing procedure and can evaluate (4) regarding it as merely a CFT correlation function on the unit disk. Then (4) is not singular and well-defined.

We may have to take conformal transformations of $(V;\Phi)$, which is singular at the point where V is inserted. For such cases, we can define $(V;\Phi)$ by taking a limit:

$$(V;\Phi) \equiv \lim_{\epsilon\to 0} \left\langle V(\epsilon,\bar{\epsilon}) \cdot f_1^{(1)} \circ \Phi(0) \right\rangle_{\text{disk}} . \tag{6}$$

Since the gauge transformation of Φ is given implicitly by (2) and is quite different from the bosonic string field theory counterpart $\delta\Phi = Q_B\Omega + \Phi * \Omega - \Omega * \Phi$, at first glance it is not clear that $(V;\Phi)$ is gauge invariant. However, for proving gauge invariance it is sufficient to show the following relation:

$$(V; A * B) = (V; B * A). \tag{7}$$

Since no explicit explanation for this relation in CFT viewpoint has not been given in the literature, we will give one shortly. In fact this is valid for any dimension (0,0) primary operator V, not necessarily BRST invariant or in small Hilbert space. In [2] this has been proved for the case where V is a tachyon vertex operator in terms of oscillator expression in flat background.

Now let us proceed assuming this relation. Then the invariance of $(V;\Phi)$ can be proved as follows. In general $\delta\Phi$ is not equal to $e^{-\Phi}\delta e^{\Phi}$, but thanks to (7), $(V;\delta\Phi) = (V; e^{-\Phi}\delta e^{\Phi})$. Then plugging (2) into this,

$$(V;\delta\Phi) \quad = \quad (V; Q_B\Omega + \eta_0\Omega'). \tag{8}$$

So far we did not use the fact that V is BRST invariant and in small Hilbert space, and by using it, we obtain

$$(V;\delta\Phi) = \left\langle \left[Q_B(V(0)\cdot f_1^{(1)}\circ\Omega(0)) + \eta_0(V(0)\cdot f_1^{(1)}\circ\Omega'(0)) \right] \right\rangle = 0. \tag{9}$$

We now make some comments on the properties of $(V;\Phi)$.

- $(V;\Phi)$ is "on-shell" in the following sense. For example, in flat background momentum q of V satisfies the on-shell condition $q^2 = \text{const.}$, because of the BRST invariance or the condition of (0,0) conformal dimension. Therefore, by momentum conservation $q+k=0$, the momentum k of Φ also satisfies $k^2 = \text{const.}$

- $(V;\Phi)$ is linear in Φ. In terms of component expression of the string field, it is surprising that all the nonlinear terms in the gauge transformation of Φ cancel.

- Since V does not have ξ_0, Φ has to have ξ_0 to obtain a nonzero contribution. Therefore we can concentrate on only those operators that survive the $\xi_0\Phi = 0$ gauge condition.

- An obvious physical interpretation of $(V;\Phi)$ is that these quantities represent couplings of one on-shell closed string mode and one open string mode.[1, 3, 2]

Now we give an explanation for the relation (7) in terms of CFT. We assume that V is a dimension (0,0) primary operator. Comparing the following two relations,

$$\begin{aligned}
\langle\langle A * B * C\rangle\rangle &= \left\langle f_1^{(3)}\circ A(0)\cdot f_2^{(3)}\circ B(0)\cdot f_3^{(3)}\circ C(0)\right\rangle_{\mathrm{disk}} \\
&= \left\langle A(0)\cdot (f_1^{(3)})^{-1}\circ\left[f_2^{(3)}\circ B(0)\cdot f_3^{(3)}\circ C(0)\right]\right\rangle \\
&= \left\langle f_1^{(2)}\circ A(0)\cdot f_1^{(2)}\circ (f_1^{(3)})^{-1}\circ\left[f_2^{(3)}\circ B(0)\cdot f_3^{(3)}\circ C(0)\right]\right\rangle \\
&= \left\langle f_1^{(2)}\circ A(0)\cdot f_2^{(2)}\circ I\circ (f_1^{(3)})^{-1}\circ\left[f_2^{(3)}\circ B(0)\cdot f_3^{(3)}\circ C(0)\right]\right\rangle, \qquad (10)\\
\langle\langle A * (B * C)\rangle\rangle &= \left\langle f_1^{(2)}\circ A(0)\cdot f_2^{(2)}\circ (B*C)(0)\right\rangle_{\mathrm{disk}}, \qquad (11)
\end{aligned}$$

we obtain the following CFT expression of $(V; A * B)$.

$$\begin{aligned}
(V;A*B) &= \lim_{\epsilon\to 0}\left\langle V(\epsilon,\bar\epsilon)\cdot f_1^{(1)}\circ I\circ (f_1^{(3)})^{-1}\circ\left[f_2^{(3)}\circ A(0)\cdot f_3^{(3)}\circ B(0)\right]\right\rangle_S \\
&= \lim_{\epsilon\to 0}\left\langle V(\epsilon,\bar\epsilon)\cdot h\circ f_1^{(1)}\circ I\circ (f_1^{(3)})^{-1}\circ\left[f_2^{(3)}\circ A(0)\cdot f_3^{(3)}\circ B(0)\right]\right\rangle_{\mathrm{disk}}, \qquad (12)
\end{aligned}$$

where $I(z) = -z^{-1}$ and $h(z) = z^{1/2}$. Let us trace the successive actions of conformal mappings in the above expression. By $f_2^{(3)}$ on A and $f_3^{(3)}$ on B, worldsheets of two open strings are glued together to form a sector with the central angle of $4\pi/3$ (fig. 1). Then by $I\circ(f_1^{(3)})^{-1}$ it is mapped to two copies of half-disk with the left half arc of one copy and the right half arc of the other identified (fig. 2). Next by $f_1^{(1)}$ they are mapped to two copies of unit disk. These two disks have cuts along the negative part of the real axis, and when we go across one of them, we jump to the other. Thus we get the Riemann surface S in fig. 3. Finally by h, $(V; A * B)$ becomes a correlation function on a unit disk. Note that V is invariant under h. At this stage we can take the limit $\epsilon\to 0$ safely and obtain the following nonsingular result:

$$(V;A*B) = \left\langle V(0,0)\cdot f_1^{(2)}\circ A(0)\cdot f_2^{(2)}\circ B(0)\right\rangle_{\mathrm{disk}}. \qquad (13)$$

We can easily see that a π rotation exchanges the positions of A and B, and leaves $V(0,0)$ invariant. (If A and B are Grassmann odd we have an extra sign factor.) Thus we have established relation (7).

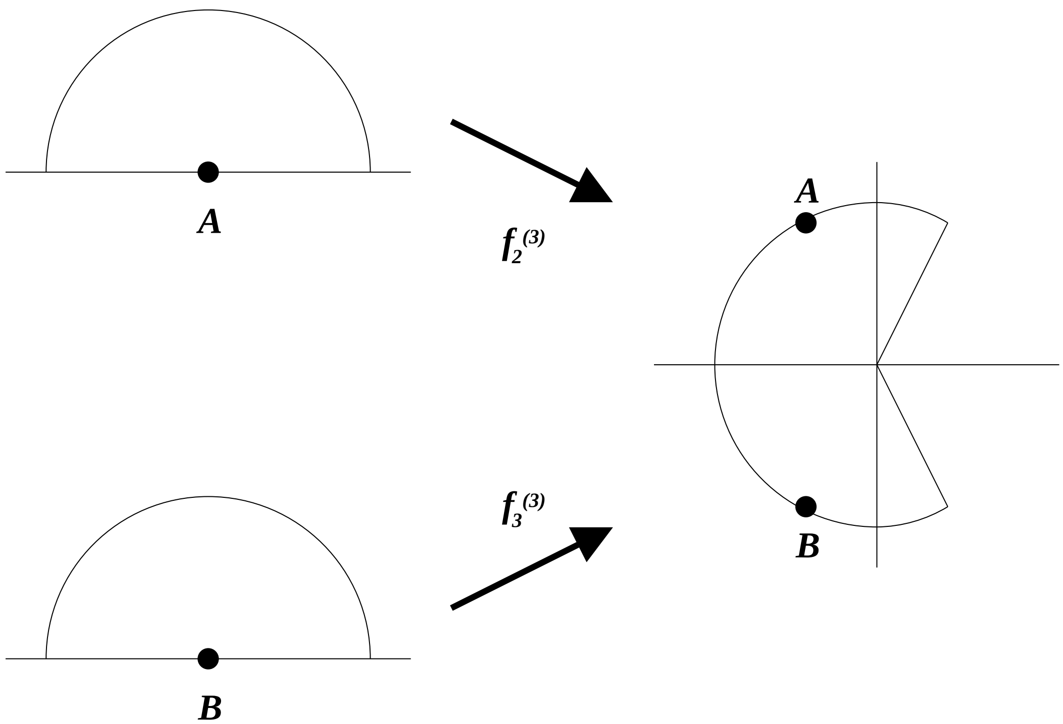

Figure 1: Worldsheets of two open strings are glued together.

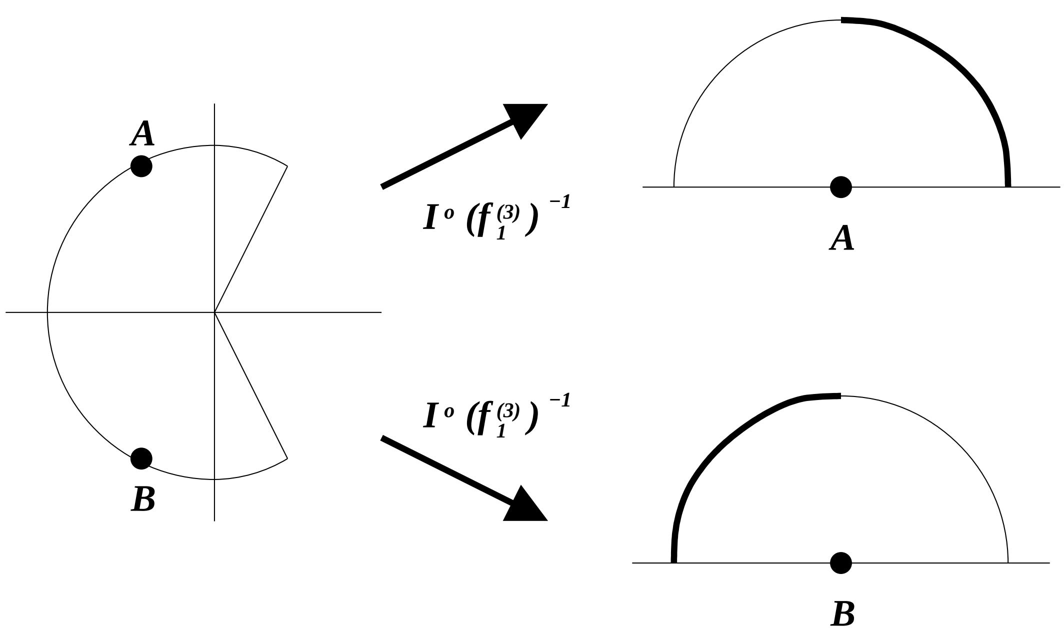

Figure 2: The sector is mapped to two copies of half disk. Identified parts are shown by bold lines.

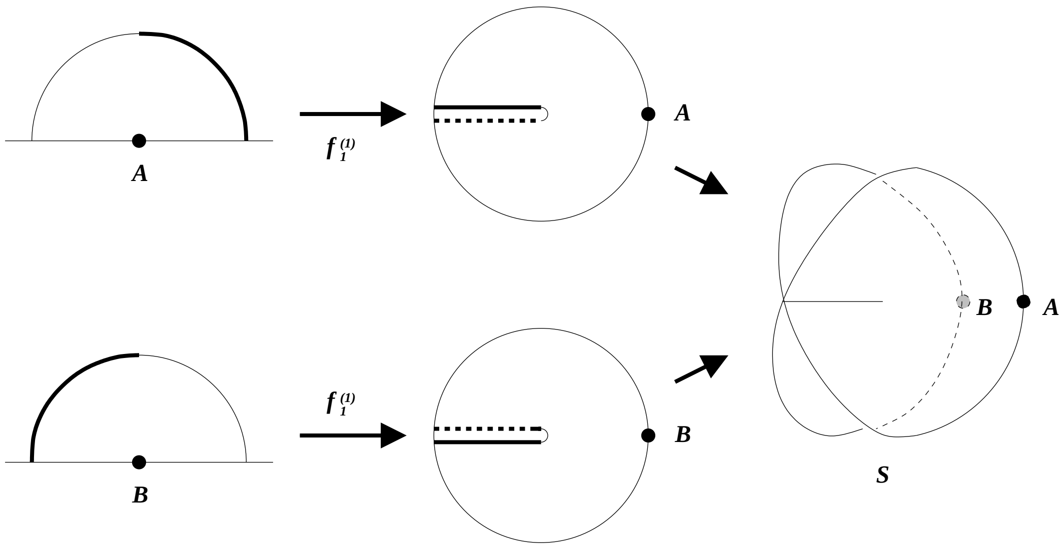

Figure 3: Two half-disks are mapped to one "double-valued" unit disk S. Identified parts are shown by bold and dashed lines.

By extending the above argument, we obtain

$$(V;\Phi_1 * \Phi_2 * \ldots * \Phi_n) = \left\langle V(0,0)\cdot f_1^{(n)}\circ\Phi_1(0)\cdot f_2^{(n)}\circ\Phi_2(0)\ldots f_n^{(n)}\circ\Phi_n(0)\right\rangle_{\text{disk}} \tag{14}$$

From this expression we see that $(V;\Phi_1 * \Phi_2 * \ldots * \Phi_n)$ is invariant under a cyclic exchange of Φ_1, $\Phi_2, \ldots$ and Φ_n.

Let us give a comment on the case where V is a dimension (h_L, h_R) primary operator. In this case V is not invariant under h and obtains the factor $(\epsilon/2)^{-h_L/2}(\bar{\epsilon}/2)^{-h_R/2}$. In general this factor is divergent or vanishing, and makes the second expression of (12) ill-defined.

3 Gauge invariant RR coupling and field strength

In this section we investigate the coupling of a string field and RR field as an application of the gauge invariant quantities defined in the previous section. From it we extract a string field theory analog of gauge invariant field strength of ordinary U(1) gauge theory up to level 2. We will see that this field strength is not gauge invariant, even on-shell.

We consider one single type IIB D9-brane in a flat background, and take RR 8-form vertex operator V_{RR8} as V. Then $(V_{RR8};\Phi)$ is interpreted as the coupling of RR 8-form and one open string field. We can expect that this corresponds to Chern–Simons term $\int C^{(8)}\wedge\widetilde{F} =$

$-\int F^{(9)} \wedge \widetilde{A}$ in the effective action, where $\widetilde{A}$ is the ordinary gauge field on the D-brane, and $\widetilde{F}$ is its field strength. Therefore we can extract a string field theory analog of $\widetilde{F}$ by computing $(V_{RR8};\Phi)$.

The lowest level component of Φ is equal to $\int \frac{d^{10}k}{(2\pi)^{10}}[A_\mu(k)\xi c\psi^\mu e^{-\phi}e^{ikX}+B(k)\xi\partial\xi c\partial c e^{-2\phi}e^{ik\cdot X}]$. In lowest order calculation, A_μ corresponds to $\widetilde{A}_\mu$ [8], and the lowest order term of $(V_{RR8};\Phi)$ should be proportional to $F_{\mu\nu}$, the field strength of A_μ. It is easy to see that this is indeed the case: $(V_{RR8};\Phi) \propto \left\langle V_{RR8}(0,0)\int \frac{d^{10}k}{(2\pi)^{10}} A_\mu(k)\xi c\psi^\mu e^{-\phi}e^{ikX}(1)\right\rangle_{\text{disk}} + \ldots$ and noting that $c\psi^\mu e^{-\phi}e^{ikX}$ is the vertex operator corresponding to the gauge field in first quantized formalism, this CFT correlator represents the coupling of RR 8-form and gauge field on D-brane in first quantized formalism, and gives $C^{(8)} \wedge F$.

Higher order terms distort the equality of A_μ and $\widetilde{A}_\mu$, as can be seen from the gauge transformation law. $\widetilde{A}_\mu$ is expected to transform as $\delta\widetilde{A}_\mu = \partial_\mu\widetilde{\lambda}$, while the transformation of A_μ contains terms of arbitrarily high order in infinitely many modes of Φ. Therefore the string theory counterpart of U(1) field strength has correction terms to F. $(V_{RR8};\Phi)$ gives a gauge invariant extension of $C^{(8)} \wedge F$, and can be expected to give some information on the correction terms.

In the following we compute $(V_{RR8};\Phi)$ order by order, up to level 2. The RR p-form vertex operator is

$$\begin{aligned} V_{RRp}(z,\bar{z}) &= F^{(p+1)}_{\mu_1\cdots\mu_{p+1}}(-q)(C\Gamma^{\mu_1\cdots\mu_{p+1}})_{AB}V_{RR}(z,\bar{z})^{AB}, \\ V_{RR}(z,\bar{z})^{AB} &= c(z)e^{-\frac{1}{2}\phi(z)}S^A(z)\tilde{c}(\bar{z})e^{-\frac{1}{2}\widetilde{\phi}(\bar{z})}\widetilde{S}^B(\bar{z})e^{-iq\cdot X}(z,\bar{z}), \end{aligned} \tag{15}$$

where $S^A(z)$ and $\widetilde{S}^B(\bar{z})$ are spin operators and, as is well known, $F^{(p+1)}_{\mu_1\cdots\mu_{p+1}}(q)$ corresponds to field strength of the RR p-form. BRST invariance of V_{RRp} requires $F^{(p+1)}(q)_{\mu_1\cdots\mu_{p+1}} = (p+1)q_{[\mu_1}C^{(p)}(q)_{\mu_2\cdots\mu_{p+1}]}$, $\quad q^{\mu_1}C^{(p)}(q)_{\mu_1\mu_2\cdots\mu_p} = 0$ and $q^2=0$. V_{RRp} also satisfies $[\eta_0, V_{RRp}] = 0$, and is a dimension (0,0) primary field.

In order to perform an order by order calculation, let us decompose Φ and Ω into sums of contributions of each level. The level is defined as $L_0 - \alpha' k^2$, i.e. the eigenvalue of L_0 with the contribution of $e^{ik\cdot X}$ subtracted.

$$\begin{aligned} \Phi &= \Phi_0 + \Phi_1 + \Phi_2 + \ldots \\ \Omega &= \Omega_0 + \Omega_1 + \Omega_2 + \ldots \end{aligned} \tag{16}$$

where Φ_n and Ω_n are level n parts of Φ and Ω respectively.

By the general argument given in the previous section, $(V_{RR8};\Phi)$ is gauge invariant. The linear part $(Q_B\Omega)+(\eta_0\Omega')$ and the higher order part in $(V_{RR8};\delta\Phi)$ cancel separately. Henceforth we consider only the linear part δ_0 of the gauge transformation, i.e. $\delta_0\Phi = Q_B\Omega$. $\eta_0\Omega'$ consists of only those without ξ_0, and does not give nonzero contribution to $(V_{RR8};\Phi)$. Therefore we do not consider the parameter Ω'.

Since Q_B does not change the level, $Q_B\Omega_n$ gives the gauge transformation of Φ_n, and we can investigate the contribution of each level separately.

First we consider the level 0 contribution. As we have already seen, $\Phi_0 = \int \frac{d^{10}k}{(2\pi)^{10}}[A_\mu(k)\xi c\psi^\mu e^{-\phi}e^{ikX} + B(k)\xi\partial\xi c\partial c e^{-2\phi}e^{ik\cdot X}]$ gives

$$(V_{RR}^{AB};\Phi_0) = -\frac{1}{\sqrt{2}}2^{2\alpha' q^2}(2i)^{\alpha' q^2/2}\left[\Gamma^\mu\frac{1}{2}(1+\Gamma^{10})C^{-1}\right]^{AB}A_\mu(q), \tag{17}$$

where we did not use $q^2=0$.

To extract a quantity analogous to the gauge field, we define $a_\mu(k)$ as follows:

$$a_\mu(q) = -\sqrt{2}\cdot 2^{-2\alpha' q^2}(2i)^{-\alpha' q^2/2}\frac{1}{16}(C\Gamma_\mu)_{AB}(V_{RR}^{AB};\Phi)_{\text{off-shell}}, \tag{18}$$

where $(V_{RR}^{AB};\Phi)_{\text{off-shell}}$ is equal to $(V_{RR}^{AB};\Phi)$ computed without using the on-shell condition $q^2=0$. In other words, $a_\mu(k)$ is the gauge field obtained by naive and straightforward off-shell extension of $(V_{RR8};\Phi)$. At level 0, $a_\mu(k) = A_\mu(k)$. Then the field strength of $a_\mu(k)$ is defined by $f_{\mu\nu}(k) = ik_\mu a_\nu(k) - ik_\nu a_\mu(k)$.

Then $(V_{RR8};\Phi)$ is given by

$$\begin{aligned}(V_{RR8};\Phi) &= -\frac{16}{\sqrt{2}}2^{2\alpha' q^2}(2i)^{\alpha' q^2/2}\epsilon^{\mu_1\dots\mu_{10}}F^{(9)}_{\mu_1\dots\mu_9}(-q)a_{\mu_{10}}(q)\\ &= -\frac{16i}{\sqrt{2}}2^{2\alpha' q^2}(2i)^{\alpha' q^2/2}\epsilon^{\mu_1\dots\mu_{10}}C^{(8)}_{\mu_1\dots\mu_8}(-q)f_{\mu_9\mu_{10}}(q).\end{aligned} \tag{19}$$

Let us see the linearized gauge transformation of $a_\mu(k)$. $\Omega_0 = \int\frac{d^{10}k}{(2\pi)^{10}}\frac{i}{\sqrt{2\alpha'}}\lambda(k)\xi\partial\xi c e^{-2\phi}e^{ikX}$ gives the following gauge transformation:

$$\delta_0 a_\mu(k) = ik_\mu\lambda(k). \tag{20}$$

At this level, a_μ transforms in the same way as ordinary gauge field. Note that the gauge transformation is defined off-shell. Therefore at this level $f_{\mu\nu}(k)$ is an off-shell gauge invariant.

Next we consider the level-1 contribution. At this level Φ_1 and Ω_1 are expanded by the basis given in the tables in the Appendix. Since V_{RR}^{AB} has two c ghosts and ϕ charge -1, and does

not have ξ and η, only those with bc ghost number 1, ϕ charge -1 and $\xi\eta$ ghost number 1 and with ξ_0 give nonzero contribution to $(V_{RR}^{AB};\Phi_1)_{\text{off-shell}}$ and a_μ. Then, the relevant basis consists of $\xi c\psi^\mu\partial e^{-\phi}e^{ik\cdot X}$, $\xi c\partial\psi^\mu e^{-\phi}e^{ik\cdot X}$, $\xi\partial c\psi^\mu e^{-\phi}e^{ik\cdot X}$, $\xi c\psi^\mu\psi^\nu\psi^\lambda e^{-\phi}e^{ik\cdot X}$ and $\xi c\psi^\mu e^{-\phi}:\partial X^\nu e^{ik\cdot X}:$. By computing the CFT correlators of these operators, we find that none of them contribute to $(V_{RR8};\Phi_1)_{\text{off-shell}}$ and a_μ.

Finally we consider level-2 contribution, and see whether $a_\mu(k)$ transforms in the same way as an ordinary gauge field. Φ_2 and Ω_2 are expanded by the basis given in the tables in the Appendix. Among them only the following components give nonzero contributions to $(V_{RR0};\Phi_2)_{\text{off-shell}}$ and $a_\mu(k)$:

$$\begin{aligned}
\Phi_2 \;=\; & \int\frac{d^{10}k}{(2\pi)^{10}}\Big[B_\mu^{(1)}(k)(\xi c\psi^\mu\partial^2 e^{-\phi}e^{ik\cdot X}) \\
& +B_\mu^{(2)}(k)(\xi c\psi^\mu:\partial^2\phi e^{-\phi}:e^{ik\cdot X}) \\
& +B_\mu^{(3)}(k)(\xi\partial^2 c\psi^\mu e^{-\phi}e^{ik\cdot X}) \\
& +B_{\mu\nu}^{(4)}(k)(\xi c\psi^\mu e^{-\phi}:\partial^2 X^\nu e^{ik\cdot X}:) \\
& +B_\mu^{(5)}(k)(\xi c\partial^2\psi^\mu e^{-\phi}e^{ik\cdot X}) \\
& +B_{\mu\nu\lambda}^{(6)}(k)(\xi c:\partial\psi^\mu\psi^\nu\psi^\lambda:e^{-\phi}e^{ik\cdot X}) \\
& +B_\mu^{(7)}(k)(:\xi\partial\xi\eta:c\psi^\mu e^{-\phi}e^{ik\cdot X}) \\
& +B_{\mu\nu\lambda}^{(8)}(k)(\xi c\psi^\mu e^{-\phi}:\partial X^\nu\partial X^\lambda e^{ik\cdot X}:) \\
& +B_\mu^{(9)}(k)(\xi:bc\partial c:\psi^\mu e^{-\phi}e^{ik\cdot X})\Big],
\end{aligned} \tag{21}$$

where the coefficients $B^{(n)}$ have appropriate symmetry, i.e. $B_{\mu\nu\lambda}^{(6)}$ is antisymmetric under the exchange of ν and λ, and $B_{\mu\nu\lambda}^{(8)}$ is symmetric under the exchange of ν and λ.

Ω_2 is expanded, and coefficients of components are defined as follows:

$$\begin{aligned}
\Omega_2 \;=\; & \int\frac{d^{10}k}{(2\pi)^{10}}\Big[\epsilon^{(1)}(k)(\xi\partial^2\xi c\partial e^{-2\phi}e^{ik\cdot X})+\epsilon^{(2)}(k)(\xi\partial\xi c\partial^2 e^{-2\phi}e^{ik\cdot X}) \\
& +\epsilon^{(3)}(k)(\xi\partial\xi c:\partial^2\phi e^{-2\phi}:e^{ik\cdot X})+\epsilon^{(4)}(k)(\xi\partial^3\xi c e^{-2\phi}e^{ik\cdot X}) \\
& +\epsilon_\mu^{(5)}(k)(\xi\partial^2\xi c e^{-2\phi}:\partial X^\mu e^{ik\cdot X}:)+\epsilon_\mu^{(6)}(k)(\xi\partial\xi c\partial e^{-2\phi}:\partial X^\mu e^{ik\cdot X}:) \\
& +\epsilon_{\mu\nu}^{(7)}(k)(\xi\partial^2\xi c:\psi^\mu\psi^\nu:e^{-2\phi}e^{ik\cdot X})+\epsilon_{\mu\nu}^{(8)}(k)(\xi\partial\xi c:\psi^\mu\psi^\nu:\partial e^{-2\phi}e^{ik\cdot X}) \\
& +\epsilon_\mu^{(9)}(k)(\xi\partial\xi\partial^2\xi c\partial^2 c\psi^\mu e^{-3\phi}e^{ik\cdot X})+\epsilon_\mu^{(10)}(k)(\xi\partial\xi c e^{-2\phi}:\partial^2 X^\mu e^{ik\cdot X}:) \\
& +\epsilon_{\mu\nu}^{(11)}(k)(\xi\partial\xi c e^{-2\phi}:\partial X^\mu\partial X^\nu e^{ik\cdot X}:)+\epsilon_{\mu\nu}^{(12)}(k)(\xi\partial\xi c:\psi^\mu\psi^\nu:e^{-2\phi}:\partial X^\lambda e^{ik\cdot X}:) \\
& +\epsilon_{\mu\nu}^{(13)}(k)(\xi\partial\xi c:\partial\psi^\mu\psi^\nu:e^{-2\phi}e^{ik\cdot X})+\epsilon_{\mu\nu\lambda\rho}^{(14)}(k)(\xi\partial\xi c:\psi^\mu\psi^\nu\psi^\lambda\psi^\rho:e^{-2\phi}e^{ik\cdot X}) \\
& +\epsilon_\mu^{(15)}(k)(\xi\psi^\mu\partial e^{-\phi}e^{ik\cdot X})+\epsilon^{(16)}(k)(\xi\partial\xi\partial^2 c e^{-2\phi}e^{ik\cdot X})
\end{aligned}$$

$$
\begin{aligned}
&+\epsilon^{(17)}_{\mu}(k)(\xi:bc:\psi^{\mu}e^{-\phi}e^{ik\cdot X})+\epsilon^{(18)}_{\mu}(k)(\xi\partial\psi^{\mu}e^{-\phi}e^{ik\cdot X})\\
&+\epsilon^{(19)}_{\mu\nu}(k)(\xi\psi^{\mu}e^{-\phi}:\partial X^{\nu}e^{ik\cdot X}:)+\epsilon^{(20)}_{\mu\nu\lambda}(k)(\xi:\psi^{\mu}\psi^{\nu}\psi^{\lambda}:e^{-\phi}e^{ik\cdot X})\\
&+\epsilon^{(21)}_{\mu}(k)(\xi\partial\xi\partial^{2}\xi c\partial c\psi^{\mu}\partial e^{-3\phi}e^{ik\cdot X})+\epsilon^{(22)}_{\mu}(k)(\xi\partial\xi\partial^{3}\xi c\partial c\psi^{\mu}e^{-3\phi}e^{ik\cdot X})\\
&+\epsilon^{(23)}(k)(\xi\partial\xi\partial^{2}\xi\partial^{3}\xi c\partial c\partial^{2}ce^{-4\phi}e^{ik\cdot X})+\epsilon^{(24)}_{\mu}(\xi\partial\xi\partial^{2}\xi c\partial c\partial\psi^{\mu}e^{-3\phi}e^{ik\cdot X})\\
&+\epsilon^{(25)}_{\mu\nu\lambda}(k)(\xi\partial\xi\partial^{2}\xi c\partial c:\psi^{\mu}\psi^{\nu}\psi^{\lambda}:e^{-3\phi}e^{ik\cdot X})+\epsilon^{(26)}_{\mu\nu}(k)(\xi\partial\xi\partial^{2}\xi c\partial c\psi^{\mu}e^{-3\phi}:\partial X^{\nu}e^{ik\cdot X}:)\\
&+\epsilon^{(27)}(k)(\xi\partial^{2}\xi\partial ce^{-2\phi}e^{ik\cdot X})+\epsilon^{(28)}(k)(\xi\partial\xi\partial c\partial e^{-2\phi}e^{ik\cdot X})\\
&+\epsilon^{(29)}(k)(\xi\partial\xi:bc\partial c:e^{-2\phi}e^{ik\cdot X})+\epsilon^{(30)}_{\mu}(k)(\xi\partial\xi\partial ce^{-2\phi}:\partial X^{\mu}e^{ik\cdot X}:)\\
&+\epsilon^{(31)}_{\mu\nu}(k)(\xi\partial\xi\partial c:\psi^{\mu}\psi^{\nu}:e^{-2\phi}e^{ik\cdot X})+\epsilon^{(32)}(k)(be^{ik\cdot X})\Big].
\end{aligned}
\tag{22}
$$

Again $\epsilon^{(n)}$ have the appropriate symmetry. Then we can compute a_μ and the gauge transformation of $B^{(n)}$:

$$
\begin{aligned}
a_{\mu}(k) &= A_{\mu}(k)-B^{(1)}_{\mu}(k)+2B^{(3)}_{\mu}(k)+2i\alpha'k^{\nu}B^{(4)}_{\mu\nu}(k)-B^{(5)}_{\mu}(k)\\
&\quad+3B^{(6)}_{\nu\mu}{}^{\nu}(k)-B^{(7)}_{\mu}(k)-\frac{1}{2}\alpha'B^{(8)}_{\mu\nu}{}^{\nu}(k),
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
\delta_0B^{(1)}_{\mu}(k) &= -6\sqrt{2\alpha'}k_{\mu}\epsilon^{(1)}(k)+13\sqrt{2\alpha'}k_{\mu}\epsilon^{(2)}(k)+\sqrt{\frac{\alpha'}{2}}k_{\mu}\epsilon^{(3)}(k)+3\sqrt{2\alpha'}k_{\mu}\epsilon^{(4)}(k)\\
&\quad-i\sqrt{\frac{\alpha'}{2}}\epsilon^{(5)}_{\mu}(k)+3i\sqrt{\frac{\alpha'}{2}}\epsilon^{(6)}_{\mu}(k)-2\sqrt{2\alpha'}k^{\nu}\epsilon^{(7)}_{\mu\nu}(k)+6\sqrt{2\alpha'}k^{\nu}\epsilon^{(8)}_{\mu\nu}(k)+8\epsilon^{(9)}_{\mu}(k)\\
&\quad-i\sqrt{\frac{\alpha'}{2}}\epsilon^{(10)}_{\mu}(k)+i\sqrt{\frac{\alpha'}{2}}\epsilon^{(12)\nu}_{\mu\nu}(k)+\sqrt{\frac{\alpha'}{2}}k^{\nu}\epsilon^{(13)}_{\nu\mu}(k)+\epsilon^{(15)}_{\mu}(k)+\frac{1}{2}\epsilon^{(17)}_{\mu}(k)\\
&\quad+36\epsilon^{(21)}_{\mu}(k)-24\epsilon^{(22)}_{\mu}(k),
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
\delta_0B^{(2)}_{\mu}(k) &= -8\sqrt{2\alpha'}k_{\mu}\epsilon^{(1)}(k)+12\sqrt{2\alpha'}k_{\mu}\epsilon^{(2)}(k)+2\sqrt{2\alpha'}k_{\mu}\epsilon^{(3)}(k)+6\sqrt{2\alpha'}k_{\mu}\epsilon^{(4)}(k)\\
&\quad-i\sqrt{2\alpha'}\epsilon^{(5)}_{\mu}(k)+2i\sqrt{2\alpha'}\epsilon^{(6)}_{\mu}(k)-4\sqrt{2\alpha'}k^{\nu}\epsilon^{(7)}_{\mu\nu}(k)+8\sqrt{2\alpha'}k^{\nu}\epsilon^{(8)}_{\mu\nu}(k)+12\epsilon^{(9)}_{\mu}(k)\\
&\quad-i\sqrt{2\alpha'}\epsilon^{(10)}_{\mu}(k)+i\sqrt{2\alpha'}\epsilon^{(12)\nu}_{\mu\nu}(k)+\sqrt{2\alpha'}k^{\nu}\epsilon^{(13)}_{\nu\mu}(k)+\frac{3}{2}\epsilon^{(17)}_{\mu}(k)\\
&\quad+48\epsilon^{(21)}_{\mu}(k)-36\epsilon^{(22)}_{\mu}(k),
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
\delta_0B^{(3)}_{\mu}(k) &= -2\epsilon^{(9)}_{\mu}(k)+\frac{1}{2}\epsilon^{(15)}_{\mu}(k)-\sqrt{2\alpha'}k_{\mu}\epsilon^{(16)}\\
&\quad-\frac{3}{2}\epsilon^{(17)}_{\mu}(k)-\frac{1}{2}\epsilon^{(18)}_{\mu}(k)-i\frac{\alpha'}{2}k^{\nu}\epsilon^{(19)}_{\mu\nu}(k),
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\delta_0B^{(4)}_{\mu\nu}(k) &= -4i\sqrt{\frac{2}{\alpha'}}\eta_{\mu\nu}\epsilon^{(1)}(k)+2i\sqrt{\frac{2}{\alpha'}}\eta_{\mu\nu}\epsilon^{(2)}(k)+i\sqrt{\frac{2}{\alpha'}}\eta_{\mu\nu}\epsilon^{(3)}(k)+6i\sqrt{\frac{2}{\alpha'}}\eta_{\mu\nu}\epsilon^{(4)}(k)\\
&\quad-4i\sqrt{\frac{2}{\alpha'}}\epsilon^{(7)}_{\mu\nu}(k)+4i\sqrt{\frac{2}{\alpha'}}\epsilon^{(8)}_{\mu\nu}(k)+\sqrt{2\alpha'}k_{\mu}\epsilon^{(10)}_{\nu}(k)+i\sqrt{\frac{2}{\alpha'}}\epsilon^{(13)}_{\nu\mu}(k)
\end{aligned}
$$

$$-2ik_\nu\epsilon^{(17)}_\mu(k)+\epsilon^{(19)}_{\mu\nu}(k), \tag{27}$$

$$\begin{aligned}
\delta_0 B^{(5)}_{\mu\nu}(k) &= -2\sqrt{2\alpha'}k_\mu\epsilon^{(1)}(k)+\sqrt{2\alpha'}k_\mu\epsilon^{(2)}(k)+\sqrt{\frac{\alpha'}{2}}k_\mu\epsilon^{(3)}(k)+3\sqrt{2\alpha'}k_\mu\epsilon^{(4)}(k)\\
&\quad -i\sqrt{\frac{\alpha'}{2}}\epsilon^{(5)}_\mu(k)+i\sqrt{\frac{\alpha'}{2}}\epsilon^{(6)}_\mu(k)-i\sqrt{\frac{\alpha'}{2}}\epsilon^{(10)}_\mu(k)-\frac{3}{4}\epsilon^{(17)}_\mu(k)-\epsilon^{(18)}_\mu(k), \qquad (28)\\
\delta_0 B^{(6)}_{\mu\nu\lambda}(k) &= 2\sqrt{2\alpha'}k_\mu\epsilon^{(7)}_{\nu\lambda}(k)-2\sqrt{2\alpha'}k_\mu\epsilon^{(8)}_{\nu\lambda}(k)-i\sqrt{\frac{\alpha'}{2}}\epsilon^{(12)}_{\nu\lambda\mu}(k)\\
&\quad +\sqrt{2\alpha'}\epsilon^{(13)}_{\mu[\nu}(k)k_{\lambda]}-\frac{1}{2}\eta_{\mu[\nu}\epsilon^{(17)}_{\lambda]}(k)+3\epsilon^{(20)}_{\mu\nu\lambda}(k), \qquad (29)\\
\delta_0 B^{(7)}_{\mu}(k) &= -2\sqrt{2\alpha'}k_\mu\epsilon^{(2)}(k)-\sqrt{2\alpha'}k_\mu\epsilon^{(3)}(k)-i\sqrt{2\alpha'}\epsilon^{(6)}_\mu(k)-4\sqrt{2\alpha'}k^\nu\epsilon^{(8)}_{\mu\nu}(k)\\
&\quad -12\epsilon^{(9)}_\mu(k)+i\sqrt{2\alpha'}\epsilon^{(10)}_\mu(k)-i\sqrt{2\alpha'}\epsilon^{(12)\nu}_{\mu\nu}(k)-\sqrt{2\alpha'}k^\nu\epsilon^{(13)}_{\nu\mu}(k)-\epsilon^{(17)}_\mu(k)\\
&\quad -36\epsilon^{(21)}_\mu(k)+24\epsilon^{(22)}_\mu(k), \qquad (30)\\
\delta_0 B^{(8)}_{\mu\nu\lambda}(k) &= 2i\sqrt{\frac{2}{\alpha'}}\eta_{\mu(\nu}\epsilon^{(5)}_{\lambda)}(k)-2i\sqrt{\frac{2}{\alpha'}}\eta_{\mu(\nu}\epsilon^{(6)}_{\lambda)}(k)+\sqrt{2\alpha'}k_\mu\epsilon^{(11)}_{\nu\lambda}(k)-2i\sqrt{\frac{2}{\alpha'}}\epsilon^{(12)}_{\mu(\nu\lambda)}(k)\\
&\quad +\frac{1}{\alpha'}\eta_{\nu\lambda}\epsilon^{(17)}_\mu(k)+2i\epsilon^{(19)}_{\mu(\nu}(k)k_{\lambda)}, \qquad (31)\\
\delta_0 B^{(9)}_{\mu}(k) &= -(\alpha' k^2+2)\epsilon^{(17)}_\mu(k)-12\epsilon^{(21)}_\mu(k)+12\epsilon^{(22)}_\mu(k)+\sqrt{2\alpha'}k_\mu\epsilon^{(29)}(k). \qquad (32)
\end{aligned}$$

By using the above expression of a_μ and $\delta_0 B^{(n)}$, we can calculate $\delta_0 a_\mu$.

$$\begin{aligned}
\delta_0 a_\mu(k) &= ik_\mu\Bigg[\lambda(k)-16i\sqrt{2\alpha'}\epsilon^{(1)}(k)+16i\sqrt{2\alpha'}\epsilon^{(2)}(k)+2i\sqrt{2\alpha'}\epsilon^{(3)}(k)+18i\sqrt{2\alpha'}\epsilon^{(4)}(k)\\
&\quad +(\sqrt{2\alpha'})^3k^\nu\epsilon^{(10)}_\nu(k)+i\sqrt{\frac{(\alpha')^3}{2}}\epsilon^{(11)\nu}_\nu(k)+3i\sqrt{\frac{\alpha'}{2}}\epsilon^{(13)\ \nu}_\nu(k)+2i\sqrt{2\alpha'}\epsilon^{(16)}(k)\Bigg]\\
&\quad -16\sqrt{2\alpha'}k^\nu\epsilon^{(7)}_{\nu\mu}(k)+16\sqrt{2\alpha'}k^\nu\epsilon^{(8)}_{\nu\mu}(k)+4\alpha' k^2\epsilon^{(17)}_\mu(k). \qquad (33)
\end{aligned}$$

Unfortunately $\delta_0 a_\mu$ is not in the form of an ordinary gauge transformation. Terms containing $\epsilon^{(7)}_{\nu\mu}$, $\epsilon^{(8)}_{\nu\mu}$ and $\epsilon^{(17)}_\mu$ are not proportional to k_μ.

The gauge transformation of $f_{\mu\nu}$ is given by

$$\begin{aligned}
\delta_0 f_{\mu\nu}(k) &= -16i\sqrt{2\alpha'}(k_\mu k^\lambda\epsilon^{(7)}_{\lambda\nu}(k)-k_\nu k^\lambda\epsilon^{(7)}_{\lambda\mu}(k))+16i\sqrt{2\alpha'}(k_\mu k^\lambda\epsilon^{(8)}_{\lambda\nu}(k)-k_\nu k^\lambda\epsilon^{(8)}_{\lambda\mu}(k))\\
&\quad +4i\alpha' k^2(k_\mu\epsilon^{(17)}_\nu(k)-k_\nu\epsilon^{(17)}_\mu(k)). \qquad (34)
\end{aligned}$$

Even on-shell $f_{\mu\nu}$ is not invariant. Of course $(V_{RR8};\Phi)$ is invariant, thanks to $k^{\mu_1}C^{(8)}_{\mu_1\mu_2\ldots\mu_8}=0$.

Thus we have seen that the field strength extracted from the RR coupling is not gauge invariant. Therefore it has some physical meaning only when it is coupled with the on-shell RR-field. It is not immediately clear whether we can modify the definition of a_μ so that it transforms in the same way as ordinary gauge field.

4 Gauge invariant component of the equation of motion and field strength

As a second application of our gauge invariant quantities, we consider in this section we "gauge invariant components" of the equation of motion, and extract a string field theory analog of field strength from it, up to level 1. We will find that this quantity is invariant under the linearized gauge transformation, even off-shell.

Let us begin with a more general consideration. If a function $f(\Phi)$ of Φ transforms under the gauge transformation covariantly, i.e. if $\delta f(\Phi) = [Q_B\Omega, f(\Phi)]$ or $[f(\Phi), \eta_0\Omega']$, V does not have to be BRST invariant or in small Hilbert space for gauge invariance of $(V; f(\Phi))$. What we need is only (7) and therefore the only necessary condition on V is that V is a dimension (0,0) primary operator.

As an example of $f(\Phi)$, we take $Q_B(e^{\Phi}(\eta_0 e^{-\Phi}))$. Since $Q_B(e^{\Phi}(\eta_0 e^{-\Phi})) = 0$ is the equation of motion derived from the gauge invariant action, it is obvious that this transforms covariantly, and indeed $\delta[Q_B(e^{\Phi}(\eta_0 e^{-\Phi}))] = [Q_B\Omega, Q_B(e^{\Phi}(\eta_0 e^{-\Phi}))]$. We may also take $\eta_0(e^{-\Phi}(Q_B e^{\Phi}))$, but $(V; \eta_0(e^{-\Phi}(Q_B e^{\Phi})))$ is equal to $(V; Q_B(e^{\Phi}(\eta_0 e^{-\Phi})))$ because of $\eta_0(e^{-\Phi}(Q_B e^{\Phi})) = e^{-\Phi} Q_B(e^{\Phi}(\eta_0 e^{-\Phi}))e^{\Phi}$.

In fact $(V; Q_B(e^{\Phi}(\eta_0 e^{-\Phi})))$ is a kind of gauge invariant we have considered in the previous sections. This can be shown as follows:

$$(V; Q_B(e^{\Phi}(\eta_0 e^{-\Phi}))) = -(Q_B V; e^{\Phi}(\eta_0 e^{-\Phi})). \tag{35}$$

$Q_B V$ is a dimension (0,0) primary operator. Therefore we can apply (7) to the right hand side, and we obtain

$$(V; Q_B(e^{\Phi}(\eta_0 e^{-\Phi}))) = -(Q_B V; -\eta_0\Phi) = (\eta_0 Q_B V; \Phi). \tag{36}$$

$\eta_0 Q_B V$ is BRST invariant, in small Hilbert space, and is a dimension (0,0) primary operator.

From this calculation we see that only the linear part of the equation of motion contributes to these gauge invariants. In general each coefficient of component operators of the basis in the linear part of the equation of motion is not gauge invariant. Therefore our gauge invariants give gauge invariant linear combinations of components of the linearized equation of motion. Since the linearized equation of motion is $Q_B\eta_0\Phi = 0$, at the linearized level these quantities are gauge invariant even off-shell, where off-shell means evaluating this quantity without imposing $q^2 = 0$.

In free U(1) gauge theory equation of motion is $\partial_\nu \widetilde{F}^{\mu\nu} = 0$, and the left hand side is gauge invariant. Let us consider a string field theory counterpart that reduces to $\partial_\nu \widetilde{F}^{\mu\nu}$ in lowest order, and extract a string field theory counterpart of $\widetilde{F}^{\mu\nu}$ from it. We choose $V^\mu(z,\bar{z}) = \xi(z)c(z)\psi^\mu(z)e^{-\phi}(z)e^{-2iq\cdot X}(z)$ as V, where the factor $e^{-2iq\cdot X}(z)$ has only left moving part. This is not an ordinary closed string vertex operator, but is a dimension (0,0) primary operator if $q^2 = 0$. We can see that this choice gives $\partial_\nu \widetilde{F}^{\mu\nu}$ for the level zero component Φ_0. The linear part of the equation of motion for Φ_0 is

$$Q_B\eta_0\Phi_0 = \int \frac{d^{10}k}{(2\pi)^{10}} \left[c\partial c\psi^\mu e^{-\phi}e^{ik\cdot X}[-\alpha' k^2 A_\mu(k) - \sqrt{2\alpha'}k_\mu B(k)] + \eta c e^{ik\cdot X}[\sqrt{2\alpha'}k_\mu A_\mu(k) + 2B(k)]\right]. \tag{37}$$

The coefficient of the second component in the right hand side gives an algebraic equation of motion for the auxiliary field $B(k)$. By using it, we obtain $k_\nu(ik^\mu A^\nu(k) - ik^\nu A^\mu(k)) = k_\nu F^{\mu\nu}(k) = 0$ from the first component. This is the ordinary equation of motion of free U(1) gauge theory. Then

$$\begin{aligned}(V^\mu; Q_B\eta_0\Phi_0)_{\text{off-shell}} &= -(i)^{2\alpha' q^2} 2^{\alpha' q^2}(-2i)^{\alpha' q^2}\alpha' \left[q^2 A^\mu(q) + \sqrt{\frac{2}{\alpha'}}q^\mu B(q)\right] \\ &= -(i)^{2\alpha' q^2} 2^{\alpha' q^2}(-2i)^{\alpha' q^2}\alpha' i q_\nu[iq^\mu A^\nu(q) - iq^\nu A^\mu(q)], \end{aligned} \tag{38}$$

where we keep V^μ off-shell.

Next we calculate the contribution of level-1 part Φ_1. Only operators with ξ_0 contribute to the linearized equation of motion, and their coefficients are defined as follows:

$$\begin{aligned}\Phi_1 = \int \frac{d^{10}k}{(2\pi)^{10}} \Big[& D^{(1)}_\mu(k)(\xi c\psi^\mu \partial e^{-\phi}e^{ik\cdot X}) \\ & + D^{(2)}(k)(\xi\partial\xi c\partial^2 c e^{-2\phi}e^{ik\cdot X}) \\ & + D^{(3)}(k)(\xi\eta e^{ik\cdot X}) \\ & + D^{(4)}_\mu(k)(\xi c\partial\psi^\mu e^{-\phi}e^{ik\cdot X}) \\ & + D^{(5)}_{\mu\nu\lambda}(k)(\xi c :\psi^\mu\psi^\nu\psi^\lambda: e^{-\phi}e^{ik\cdot X}) \\ & + D^{(6)}_{\mu\nu}(k)(\xi c\psi^\mu e^{-\phi} :\partial X^\nu e^{ik\cdot X}:) \\ & + D^{(7)}(k)(\xi\partial^2\xi c\partial c e^{-2\phi}e^{ik\cdot X}) \\ & + D^{(8)}(k)(\xi\partial\xi c\partial c\partial e^{-2\phi}e^{ik\cdot X}) \\ & + D^{(9)}_{\mu\nu}(k)(\xi\partial\xi c\partial c :\psi^\mu\psi^\nu: e^{-2\phi}e^{ik\cdot X}) \\ & + D^{(10)}_\mu(k)(\xi\partial\xi c\partial c e^{-2\phi} :\partial X^\mu e^{ik\cdot X}:) \\ & + D^{(11)}_\mu(k)(\xi\partial c\psi^\mu e^{-\phi}e^{ik\cdot X})\end{aligned}$$

$$+(\text{no }\xi_0\text{ part})\Big]. \tag{39}$$

The linearized equation of motion is given as follows:

$$\begin{aligned}
Q_B\eta_0\Phi_1 \;=\; & \int\frac{d^{10}k}{(2\pi)^{10}}\Big[(c\partial c\psi^\mu\partial e^{-\phi}e^{ik\cdot X})\\
&\times\Big[-(\alpha' k^2+1)D^{(1)}_\mu(k)+2\sqrt{2\alpha'}k_\mu D^{(7)}(k)-4\sqrt{2\alpha'}k_\mu D^{(8)}(k)\\
&-2\sqrt{2\alpha'}k^\nu D^{(9)}_{\mu\nu}(k)-i\sqrt{\frac{\alpha'}{2}}D^{(10)}_\mu(k)+D^{(11)}_\mu(k)\Big]\\
&+(\partial\xi c\partial c\partial^2 c e^{-2\phi}e^{ik\cdot X})\\
&\times\Big[(\alpha' k^2+1)D^{(2)}(k)-D^{(7)}(k)+i\frac{\alpha'}{2}k^\mu D^{(10)}_\mu(k)\Big]\\
&+(\eta\partial c e^{ik\cdot X})\\
&\times\Big[-(\alpha' k^2+1)D^{(3)}(k)-6D^{(7)}(k)+8D^{(8)}(k)+\sqrt{2\alpha'}k^\mu D^{(11)}_\mu(k)\Big]\\
&+(c\partial c\partial\psi^\mu e^{-\phi}e^{ik\cdot X})\\
&\times\Big[-(\alpha' k^2+1)D^{(4)}_\mu(k)-2\sqrt{2\alpha'}k_\mu D^{(7)}(k)+2\sqrt{2\alpha'}k_\mu D^{(8)}(k)+i\sqrt{\frac{\alpha'}{2}}D^{(10)}_\mu(k)+D^{(11)}_\mu(k)\Big]\\
&+(c\partial c:\psi^\mu\psi^\nu\psi^\lambda:e^{-\phi}e^{ik\cdot X})\\
&\times\Big[-(\alpha' k^2+1)D^{(5)}_{\mu\nu\lambda}(k)-\sqrt{2\alpha'}k_{[\mu}D^{(9)}_{\nu\lambda]}(k)\Big]\\
&+(c\partial c\psi^\mu e^{-\phi}:\partial X^\nu e^{ik\cdot X}:)\\
&\times\Big[-(\alpha' k^2+1)D^{(6)}_{\mu\nu}(k)-2i\sqrt{\frac{2}{\alpha'}}\eta_{\mu\nu}D^{(7)}(k)+2i\sqrt{\frac{2}{\alpha'}}\eta_{\mu\nu}D^{(8)}(k)+2i\sqrt{\frac{2}{\alpha'}}D^{(9)}_{\mu\nu}(k)\\
&-\sqrt{2\alpha'}k_\mu D^{(10)}_\nu(k)+2ik_\nu D^{(11)}_\mu(k)\Big]\\
&+(c\partial^2 c\psi^\mu e^{-\phi}e^{ik\cdot X})\\
&\times\Big[-\frac{1}{2}D^{(1)}_\mu(k)-\sqrt{2\alpha'}k_\mu D^{(2)}(k)-\frac{1}{2}D^{(4)}_\mu(k)+i\frac{\alpha'}{2}k^\nu D^{(6)}_{\mu\nu}(k)+D^{(11)}_\mu(k)\Big]\\
&+(\partial\eta c e^{ik\cdot X})\\
&\times\Big[-\sqrt{2\alpha'}k^\mu D^{(1)}_\mu(k)+6D^{(2)}(k)-D^{(3)}(k)+\sqrt{2\alpha'}k^\mu D^{(4)}_\mu(k)-i\sqrt{\frac{\alpha'}{2}}D^{(6)\mu}_\mu(k)\\
&+8D^{(7)}(k)-12D^{(8)}(k)\Big]\\
&+(\eta c\partial\phi e^{ik\cdot X})\\
&\times\Big[-2\sqrt{2\alpha'}k^\mu D^{(1)}_\mu(k)+8D^{(2)}(k)+\sqrt{2\alpha'}k^\mu D^{(4)}_\mu(k)-i\sqrt{\frac{\alpha'}{2}}D^{(6)\mu}_\mu(k)
\end{aligned}$$

$$
\begin{aligned}
&+12D^{(7)}(k) - 20D^{(8)}(k)\Big] \\
&+(\eta c : \psi^\mu \psi^\nu : e^{ik\cdot X}) \\
&\times\Big[-\sqrt{2\alpha'}k_{[\mu}D^{(1)}_{\nu]}(k) + 3\sqrt{2\alpha'}k^\lambda D^{(5)}_{\mu\nu\lambda}(k) + i\sqrt{\frac{\alpha'}{2}}D^{(6)}_{[\mu\nu]}(k) + 2D^{(9)}_{\mu\nu}(k)\Big] \\
&+(\eta c : \partial X^\mu e^{ik\cdot X} :) \\
&\times\Big[-i\sqrt{\frac{2}{\alpha'}}D^{(1)}_\mu(k) - 2ik_\mu D^{(3)}(k) + i\sqrt{\frac{2}{\alpha'}}D^{(4)}_\mu(k) + \sqrt{2\alpha'}k^\nu D^{(6)}_{\nu\mu}(k) + 2D^{(10)}_\mu(k)\Big]\Big].
\end{aligned} \tag{40}
$$

The first six components of the above equation have Klein–Gordon operator $(\alpha' k^2+1)$, and the last five components do not. Therefore the first six are dynamical equations of motion and the rest is algebraic equations for auxiliary fields $D^{(7)}$, $D^{(8)}$, $D^{(9)}_{\mu\nu}$, $D^{(10)}_\mu$, and $D^{(11)}_\mu$. This can also be seen from the fact that operators corresponding to these auxiliary fields contain c_0, and can be gauged away by taking the gauge $b_0\Phi = 0$.

Computation of $(V^\mu; Q_B\eta_0\Phi_1)$ is straightforward.

$$
\begin{aligned}
(V_\mu; Q_B\eta_0\Phi_1) \;=\;& -i(i)^{2\alpha' q^2}2^{\alpha' q^2}(-2i)^{\alpha' q^2}\Big[-2\alpha' q^2 D^{(1)}_\mu(q) + 4\sqrt{2\alpha'}q_\mu D^{(2)}(q) - 2\alpha' q^2 D^{(4)}_\mu(q) \\
&+2i(\alpha')^2 q^2 q^\nu D^{(6)}_{\mu\nu}(q) - 4\sqrt{2\alpha'}q_\mu D^{(7)}(q) + 2i\sqrt{2\alpha'}\alpha' q_\mu q^\nu D^{(10)}_\nu(q) \\
&+4\alpha' q^2 D^{(11)}_\mu(q)\Big].
\end{aligned} \tag{41}
$$

Eliminating auxiliary fields $D^{(7)}$, $D^{(10)}_\mu$, and $D^{(11)}_\mu$ by algebraic equations of motion, we obtain

$$
\begin{aligned}
(V_\mu; Q_B\eta_0\Phi_1) \;=\;& -i(i)^{2\alpha' q^2}2^{\alpha' q^2}(-2i)^{\alpha' q^2}\alpha' q^2\Big[8\sqrt{2\alpha'}(\alpha' q^2+1)q_\mu D^{(2)}(q) - 4\sqrt{2\alpha'}(\alpha' q^2+1)q_\mu D^{(3)}(q) \\
&+4\alpha'(2q_\mu q^\nu D^{(4)}_\nu(q) + q^2 D^{(4)}_\mu(q)) \\
&+2i\alpha' q^\nu(D^{(6)}_{\mu\nu}(q) + D^{(6)}_{\nu\mu}(q)) - 2i\alpha' q_\mu D^{(6)\nu}_\nu(q) - 4i(\alpha')^2 q^\mu q^\nu q^\lambda D^{(6)}_{\nu\lambda}(q)\Big].
\end{aligned} \tag{42}
$$

Thus, up to this order, we get the following quantity:

$$
\begin{aligned}
&iq_\nu[iq_\mu A^\nu(q) - iq^\nu A_\mu(q)] \\
&+q^2\Big[8\sqrt{2\alpha'}(\alpha' q^2+1)q_\mu D^{(2)}(q) - 4\sqrt{2\alpha'}(\alpha' q^2+1)q_\mu D^{(3)}(q) \\
&+4\alpha'(2q_\mu q^\nu D^{(4)}_\nu(q) + q^2 D^{(4)}_\mu(q)) \\
&+2i\alpha' q^\nu(D^{(6)}_{\mu\nu}(q) + D^{(6)}_{\nu\mu}(q)) - 2i\alpha' q_\mu D^{(6)\nu}_\nu(q) - 4i(\alpha')^2 q^\mu q^\nu q^\lambda D^{(6)}_{\nu\lambda}(q)\Big] + \ldots.
\end{aligned} \tag{43}
$$

This is gauge invariant if $q^2 = 0$, and as we have already noticed, this is invariant under a linearized gauge transformation even off-shell. This is interpreted as an extension of $q_\nu F^{\mu\nu}(q)$. However the contribution from the level-1 part is not in the form of $q_\nu(\ldots)^{[\mu\nu]}$. It is thus not possible to extract the counterpart of $F^{\mu\nu}(q)$ from this quantity.

Therefore we want to make a better choice for V. To give a gauge invariant in the form of $q_\nu(\ldots)^{[\mu\nu]}$, $(V^\mu; Q_B\eta_0\Phi)$ has to satisfy $q_\mu(V^\mu; Q_B\eta_0\Phi) = 0$. Note the following equation:

$$
\begin{aligned}
Q_B[\xi\partial\xi c e^{-2\phi} e^{-2iq\cdot X}] &= q_\mu V'^\mu - e^{-2iq\cdot X}, \qquad (44)\\
V'^\mu &= V^\mu + \sqrt{\frac{\alpha'}{2}} q^\mu \xi\partial\xi c\partial c e^{-2\phi} e^{-2iq\cdot X}. \qquad (45)
\end{aligned}
$$

This shows that if we add $\sqrt{\frac{\alpha'}{2}} q^\mu(\xi\partial\xi c\partial c e^{-2\phi} e^{-2iq\cdot X}; Q_B\eta_0\Phi)$ to $(V^\mu; Q_B\eta_0\Phi)$, we obtain a gauge invariant in the form of $q_\nu(\ldots)^{[\mu\nu]}$. This is because the second term in the right hand side of (44) does not contribute since there is no zero mode of ξ in the correlator, and when V'^μ is contracted with q_μ the whole thing in the correlator becomes BRST exact.

The component expression of $(V'^\mu; Q_B\eta_0\Phi)$ up to level 1 is given by

$$
\begin{aligned}
(V'_\mu; Q_B\eta_0\Phi) &= (i)^{2\alpha' q^2} 2^{\alpha' q^2} (-2i)^{\alpha' q^2} \alpha' \Big[q^\nu (q_\mu A_\nu - q_\nu A_\mu) - 2iq^\nu (q_\mu D^{(1)}_\nu - q_\nu D^{(1)}_\mu) \\
&\quad -2iq^\nu (q_\mu D^{(4)}_\nu - q_\nu D^{(4)}_\mu) - 2\alpha' q^\nu q_\lambda (q_\mu D^{(6)\lambda}_\nu - q_\nu D^{(6)\lambda}_\mu) + 4iq^\nu (q_\mu D^{(11)}_\nu - q_\nu D^{(11)}_\mu) \Big] \\
&= (i)^{2\alpha' q^2} 2^{\alpha' q^2} (-2i)^{\alpha' q^2} \alpha' q^\nu \Big[q_\mu (A_\nu - 2iD^{(1)}_\nu - 2iD^{(4)}_\nu - 2\alpha' q_\lambda D^{(6)\lambda}_\nu + 4iD^{(11)}_\nu) \\
&\quad - q_\nu (A_\mu - 2iD^{(1)}_\mu - 2iD^{(4)}_\mu - 2\alpha' q_\lambda D^{(6)\lambda}_\mu + 4iD^{(11)}_\mu) \Big]. \qquad (46)
\end{aligned}
$$

This is in the form of $q_\nu(\ldots)^{[\mu\nu]}$. We have thus succeeded in extracting string field theory counterparts $f_{\mu\nu}$ and a_μ of $\widetilde{F}_{\mu\nu}$ and $\widetilde{A}_\mu$, up to this level. Note that we did not use algebraic equations of motion for auxiliary fields to extract these quantities.

$$
\begin{aligned}
f_{\mu\nu}(q) &= iq_\mu a_\nu(q) - iq_\nu a_\mu(q), \\
a_\mu(q) &= A_\mu(q) - 2iD^{(1)}_\mu(q) - 2iD^{(4)}_\mu(q) - 2\alpha' q_\lambda D^{(6)\lambda}_\mu(q) + 4iD^{(11)}_\mu(q). \qquad (47)
\end{aligned}
$$

The linearized gauge transformation of $a_\mu(q)$ is

$$
\delta_0 a_\mu(q) = iq_\mu[\lambda(q) - 2i\sqrt{2\alpha'}\zeta^{(1)}(q) - \sqrt{2\alpha'}\alpha' q^\nu \zeta^{(4)}_\nu(q) + 2i\sqrt{2\alpha'}\zeta^{(7)}(q)], \qquad (48)
$$

where $\zeta^{(n)}$ are defined as:

$$
\begin{aligned}
\Omega_1 = \int \frac{d^{10}k}{(2\pi)^{10}} [&\zeta^{(1)}(k)\xi\partial^2\xi c e^{-2\phi} e^{ikX} \\
&+\zeta^{(2)}(k)\xi\partial\xi c\partial e^{-2\phi} e^{ikX} \\
&+\zeta^{(3)}_{\mu\nu}(k)\xi\partial\xi c : \psi^\mu\psi^\nu : e^{-2\phi} e^{ikX} \\
&+\zeta^{(4)}_\mu(k)\xi\partial\xi c e^{-2\phi} : \partial X^\mu e^{ikX} : \\
&+\zeta^{(5)}_\mu(k)\xi\psi^\mu e^{-\phi} e^{ikX} \\
&+\zeta^{(6)}_\mu(k)\xi\partial\xi\partial^2\xi c\partial c\psi^\mu e^{-3\phi} e^{ikX} \\
&+\zeta^{(7)}(k)\xi\partial\xi\partial c e^{-2\phi} e^{ikX}], \qquad (49)
\end{aligned}
$$

and the linearized gauge transformation for each component is

$$
\begin{aligned}
\delta_0 D^{(1)}_\mu(k) &= -2\sqrt{2\alpha'}k_\mu\zeta^{(1)}(k) + 4\sqrt{2\alpha'}k_\mu\zeta^{(2)}(k) + 2\sqrt{2\alpha'}k^\nu\zeta^{(3)}_{\mu\nu}(k) + i\sqrt{\frac{\alpha'}{2}}\zeta^{(4)}_\mu(k) \\
&\quad -\zeta^{(5)}_\mu(k) + 4\zeta^{(6)}_\mu(k), && (50)\\
\delta_0 D^{(2)}(k) &= -\zeta^{(1)}(k) + i\frac{\alpha'}{2}k^\mu\zeta^{(4)}_\mu(k) + \zeta^{(7)}(k), && (51)\\
\delta_0 D^{(3)}(k) &= 6\zeta^{(1)}(k) - 8\zeta^{(2)}(k) - \sqrt{2\alpha'}k^\mu\zeta^{(5)}_\mu(k) + 2\zeta^{(7)}(k), && (52)\\
\delta_0 D^{(4)}_\mu(k) &= 2\sqrt{2\alpha'}k_\mu\zeta^{(1)}(k) - 2\sqrt{2\alpha'}k_\mu\zeta^{(2)}(k) - i\sqrt{\frac{\alpha'}{2}}\zeta^{(4)}_\mu(k) - \zeta^{(5)}_\mu(k), && (53)\\
\delta_0 D^{(5)}_{\mu\nu\lambda}(k) &= \sqrt{2\alpha'}k_{[\mu}\zeta^{(3)}_{\nu\lambda]}(k), && (54)\\
\delta_0 D^{(6)}_{\mu\nu}(k) &= 2i\sqrt{\frac{2}{\alpha'}}\eta_{\mu\nu}\zeta^{(1)}(k) - 2i\sqrt{\frac{2}{\alpha'}}\eta_{\mu\nu}\zeta^{(2)}(k) - 2i\sqrt{\frac{2}{\alpha'}}\zeta^{(3)}_{\mu\nu}(k) \\
&\quad +\sqrt{2\alpha'}k_\mu\zeta^{(4)}_\nu(k) - 2ik_\nu\zeta^{(5)}_\mu(k), && (55)\\
\delta_0 D^{(7)}(k) &= -(\alpha' k^2+1)\zeta^{(1)}(k) - \sqrt{2\alpha'}k^\mu\zeta^{(6)}_\mu(k) + \zeta^{(7)}(k), && (56)\\
\delta_0 D^{(8)}(k) &= -(\alpha' k^2+1)\zeta^{(2)}(k) - \sqrt{2\alpha'}k^\mu\zeta^{(6)}_\mu(k), +\zeta^{(7)}(k), && (57)\\
\delta_0 D^{(9)}_{\mu\nu}(k) &= -(\alpha' k^2+1)\zeta^{(3)}_{\mu\nu}(k) + 2\sqrt{2\alpha'}k_{[\mu}\zeta^{(6)}_{\nu]}(k), && (58)\\
\delta_0 D^{(10)}_\mu(k) &= -(\alpha' k^2+1)\zeta^{(4)}_\mu(k) + 2i\sqrt{\frac{2}{\alpha'}}\zeta^{(6)}_\mu(k) + 2ik_\mu\zeta^{(7)}(k), && (59)\\
\delta_0 D^{(11)}_\mu(k) &= -(\alpha' k^2+1)\zeta^{(5)}_\mu(k) + 2\zeta^{(6)}_\mu(k) + \sqrt{2\alpha'}k_\mu\zeta^{(7)}(k). && (60)
\end{aligned}
$$

V'_μ is one of the simplest choices for V, but of course we can take different ones. For example,

$$
\begin{aligned}
V''^\mu(z,\bar z) &= \xi(z)c(z)\psi^\mu(z)e^{-\phi}(z)e^{-iq\cdot X}(z,\bar z) \\
&\quad +\eta(\bar z)\xi(z)\partial\xi(z)c(z)e^{\phi}(\bar z)e^{-2\phi}(z)\psi^\mu(\bar z)e^{-iq\cdot X}(z,\bar z) \\
&\quad +\frac{1}{2}\sqrt{\frac{\alpha'}{2}}q^\mu\xi(z)\partial\xi(z)c(z)(\partial c(z)+\partial c(\bar z))e^{-2\phi}(z)e^{-iq\cdot X}(z,\bar z) \\
&\quad -i\sqrt{\frac{2}{\alpha'}}\xi(z)\partial\xi(z)c(z)c(\bar z)e^{-2\phi}(z) :\partial X^\mu(\bar z)e^{-iq\cdot X}(z,\bar z):, \qquad (61)
\end{aligned}
$$

where $e^{-iq\cdot X}(z,\bar z)$ is an ordinary momentum factor for closed strings, and the normal ordering of $:\partial X^\mu(\bar z)e^{-iq\cdot X}(z,\bar z):$ acts on left mover and right mover separately. This satisfies $Q_B[\xi(z)\partial\xi(z)c(z)e^{-2\phi}(z)e^{-iq\cdot X}(z,\bar z)] = -\sqrt{\frac{\alpha'}{2}}q_\mu V''^\mu(z,\bar z) - e^{-iq\cdot X}(z,\bar z)$. This equation shows that $V''^\mu(z,\bar z)$ is one of the right choices for V.

We have considered only the linearized part of the gauge transformation. Some choices for

V might give gauge invariant quantities under full gauge transformation even off-shell, and others might not.

5 Discussion

We have considered "on-shell" gauge invariants in Berkovits' open superstring field theory and, as an application of these we have computed a string field theory analog of RR coupling $\int C^{(8)} \wedge \widetilde{F}$ up to level 2, and a gauge invariant component of the equation of motion up to level 1. We have extracted analogs of the field strength from them. The former is not gauge invariant even on-shell, and the corresponding gauge field a_μ does not transform as an ordinary gauge field. The latter is gauge invariant under full gauge transformation if on-shell, and invariant under linearized gauge transformation even off-shell.

It seems that the latter is more promising, but obviously there are many open problems. First of all, it is not clear whether it is fully invariant off-shell for some choices of V and, if not, whether there is a way of extending it to off-shell. By momentum conservation we always have some kind of on-shell condition if we use on-shell closed string vertex operators, or dimension (0,0) operators.

Another unclear point is what our gauge invariant gives after integrating out massive modes, and how it is related to the gauge field given in [9, 10] and marginal deformation in [11, 12].

The construction of field strength in the nonabelian case, i.e. multiple D-brane case, is an interesting direction of extension, although our method is not directly applicable to this case. For RR coupling, the nonabelian part of the field strength does not couple to the RR 8-form. More technically, relation (7) does not hold for matrix valued string fields.

Acknowledgments

This work is supported by the Japan Society for the Promotion of Science and the Swiss National Science Foundation.

Appendix

In this appendix we tabulate the bases for expanding the string field Φ and the gauge trans-

formation parameter Ω. The following two vacua are used in the tables:

$$|\Omega\rangle = c_1 e^{ik\cdot X(0)} |0\rangle\,, \quad \left|\widetilde{\Omega}\right\rangle = e^{-\phi}(0) c_1 e^{ik\cdot X(0)} |0\rangle\,, \tag{62}$$

where $|0\rangle$ is Fock vacuum defined by $\alpha^{\mu}_{n\geq 1}|0\rangle = \psi^{\mu}_{n\geq 1/2}|0\rangle = b_{n\geq -1}|0\rangle = c_{n\geq 2}|0\rangle = \beta_{n\geq -1/2}|0\rangle = \gamma_{n\geq 3/2}|0\rangle = 0$. We show both oscillator expressions and operator expressions. Oscillator expression $|\text{osc.}\rangle$ and the corresponding operator expression v is related by $|\text{osc.}\rangle = (\text{numerical factor})v(0)|0\rangle$.

Level	Oscillator expression	Operator expression
0	$\xi_0\psi^{\mu}_{-1/2}\left\|\widetilde{\Omega}\right\rangle$	$\xi c\psi^{\mu}e^{-\phi}e^{2ik\cdot X}$
1	$\xi_0\beta_{-1/2}\gamma_{-1/2}\psi^{\mu}_{-1/2}\left\|\widetilde{\Omega}\right\rangle$	$\xi c\psi^{\mu}\partial e^{-\phi}e^{2ik\cdot X}$
	$\xi_0\beta_{-1/2}c_{-1}\left\|\widetilde{\Omega}\right\rangle$	$\xi\partial\xi c\partial^2 c e^{-2\phi}e^{2ik\cdot X}$
	$\xi_0\gamma_{-1/2}b_{-1}\left\|\widetilde{\Omega}\right\rangle$	$:\xi\eta:e^{2ik\cdot X}$
	$\xi_0\psi^{\mu}_{-3/2}\left\|\widetilde{\Omega}\right\rangle$	$\xi c\partial\psi^{\mu}e^{-\phi}e^{2ik\cdot X}$
	$\xi_0\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}\psi^{\lambda}_{-1/2}\left\|\widetilde{\Omega}\right\rangle$	$\xi c:\psi^{\mu}\psi^{\nu}\psi^{\lambda}:e^{-\phi}e^{2ik\cdot X}$
	$\xi_0\psi^{\mu}_{-1/2}\alpha^{\nu}_{-1}\left\|\widetilde{\Omega}\right\rangle$	$\xi c\psi^{\mu}e^{-\phi}:\partial X^{\nu}e^{2ik\cdot X}:$
2	$\xi_0\psi^{\mu}_{-5/2}\left\|\widetilde{\Omega}\right\rangle$	$\xi c\psi^{\mu}\partial^2 e^{-\phi}e^{ikX}$
	$\xi_0\beta_{-3/2}\gamma_{-1/2}\psi^{\mu}_{-1/2}\left\|\widetilde{\Omega}\right\rangle$	$(\xi c\partial^2 e^{-\phi} - 2:\eta\xi\partial\xi:ce^{-\phi})\psi^{\mu}e^{ikX}$
	$\xi_0\gamma_{-3/2}\beta_{-1/2}\psi^{\mu}_{-1/2}\left\|\widetilde{\Omega}\right\rangle$	$(\xi c(\partial\phi\partial\phi + \partial^2\phi)e^{-\phi} - 2:\eta\xi\partial\xi:ce^{-\phi})\psi^{\mu}e^{ikX}$
	$\xi_0\psi^{\mu}_{-3/2}\beta_{-1/2}\gamma_{-1/2}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\psi^{\mu}_{-3/2}\psi^{\nu}_{-1/2}\psi^{\lambda}_{-1/2}\left\|\widetilde{\Omega}\right\rangle$	$\xi c:\partial\psi^{\mu}\psi^{\nu}\psi^{\lambda}:e^{-\phi}e^{ikX}$
	$\xi_0\beta_{-3/2}c_{-1}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\gamma_{-3/2}b_{-1}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\psi^{\mu}_{-3/2}\alpha^{\nu}_{-1}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\beta_{-1/2}\beta_{-1/2}\gamma_{-1/2}\gamma_{-1/2}\psi^{\mu}_{-1/2}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\beta_{-1/2}\gamma_{-1/2}\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}\psi^{\lambda}_{-1/2}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}\psi^{\lambda}_{-1/2}\psi^{\rho}_{-1/2}\psi^{\sigma}_{-1/2}\left\|\widetilde{\Omega}\right\rangle$	$\xi c:\psi^{\mu}\psi^{\nu}\psi^{\lambda}\psi^{\rho}\psi^{\sigma}:e^{-\phi}e^{ikX}$
	$\xi_0\beta_{-1/2}\beta_{-1/2}\gamma_{-1/2}c_{-1}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\beta_{-1/2}\gamma_{-1/2}\gamma_{-1/2}b_{-1}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\beta_{-1/2}\gamma_{-1/2}\psi^{\mu}_{-1/2}\alpha^{\nu}_{-1}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\beta_{-1/2}\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}c_{-1}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\gamma_{-1/2}\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}b_{-1}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}\psi^{\lambda}_{-1/2}\alpha^{\rho}_{-1}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\beta_{-1/2}c_{-2}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\gamma_{-1/2}b_{-2}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\psi^{\mu}_{-1/2}\alpha^{\nu}_{-2}\left\|\widetilde{\Omega}\right\rangle$	$\xi c\psi^{\mu}e^{-\phi}:\partial^2 X^{\nu}e^{ikX}:$
	$\xi_0\beta_{-1/2}c_{-1}\alpha^{\mu}_{-1}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\gamma_{-1/2}b_{-1}\alpha^{\mu}_{-1}\left\|\widetilde{\Omega}\right\rangle$	
	$\xi_0\psi^{\mu}_{-1/2}\alpha^{\nu}_{-1}\alpha^{\lambda}_{-1}\left\|\widetilde{\Omega}\right\rangle$	$\xi c\psi^{\mu}e^{-\phi}:\partial X^{\nu}\partial X^{\lambda}e^{ikX}:$
	$\xi_0\psi^{\mu}_{-1/2}b_{-1}c_{-1}\left\|\widetilde{\Omega}\right\rangle$	$\xi\partial^2 c\psi^{\mu}e^{-\phi}e^{ikX}$

Table 1: Basis for Φ. Only those with ξ_0 and without c_0 are shown. For the level-2 basis, the operator expressions are given only for those relevant to our calculation.

Level	Oscillator expression	Operator expression
0	$\xi_0 c_0 \beta_{-1/2} \vert\tilde{\Omega}\rangle$	$\xi\partial\xi c\partial c e^{-2\phi} e^{2ik\cdot X}$
1	$\xi_0 c_0 \beta_{-3/2} \vert\tilde{\Omega}\rangle$	$(\xi\partial^2\xi c\partial c e^{-2\phi} + \frac{1}{2}\xi\partial\xi c\partial c\partial e^{-2\phi}) e^{2ik\cdot X}$
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2}\gamma_{-1/2} \vert\tilde{\Omega}\rangle$	$(\xi\partial^2\xi c\partial c e^{-2\phi} + \xi\partial\xi c\partial c\partial e^{-2\phi}) e^{2ik\cdot X}$
	$\xi_0 c_0 \beta_{-1/2}\psi^\mu_{-1/2}\psi^\nu_{-1/2} \vert\tilde{\Omega}\rangle$	$\xi\partial\xi c\partial c : \psi^\mu\psi^\nu : e^{-2\phi} e^{2ik\cdot X}$
	$\xi_0 c_0 \beta_{-1/2}\alpha^\mu_{-1} \vert\tilde{\Omega}\rangle$	$\xi\partial\xi c\partial c e^{-2\phi} : \partial X^\mu e^{2ik\cdot X} :$
	$\xi_0 c_0 \psi^\mu_{-1/2} b_{-1} \vert\tilde{\Omega}\rangle$	$\xi\partial c\psi^\mu e^{-\phi} e^{2ik\cdot X}$
2	$\xi_0 c_0 \beta_{-5/2} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-3/2}\psi^\mu_{-1/2}\psi^\nu_{-1/2} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-3/2}\beta_{-1/2}\gamma_{-1/2} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \gamma_{-3/2}\beta_{-1/2}\beta_{-1/2} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \psi^\mu_{-3/2}\beta_{-1/2}\psi^\nu_{-1/2} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-3/2}\alpha^\mu_{-1} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \psi^\mu_{-3/2} b_{-1} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2}\beta_{-1/2}\gamma_{-1/2}\gamma_{-1/2} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2}\gamma_{-1/2}\psi^\mu_{-1/2}\psi^\nu_{-1/2} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-1/2}\psi^\mu_{-1/2}\psi^\nu_{-1/2}\psi^\lambda_{-1/2}\psi^\rho_{-1/2} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2}\gamma_{-1/2}\alpha^\mu_{-1} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2}\psi^\mu_{-1/2} c_{-1} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-1/2}\gamma_{-1/2}\psi^\mu_{-1/2} b_{-1} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-1/2}\psi^\mu_{-1/2}\psi^\nu_{-1/2}\alpha^\lambda_{-1} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \psi^\mu_{-1/2}\psi^\nu_{-1/2}\psi^\lambda_{-1/2} b_{-1} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-1/2}\alpha^\mu_{-2} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \psi^\mu_{-1/2} b_{-2} \vert\tilde{\Omega}\rangle$	$\xi : bc\partial c : \psi^\mu e^{-\phi} e^{ikX}$
	$\xi_0 c_0 \beta_{-1/2}\alpha^\mu_{-1}\alpha^\nu_{-1} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \beta_{-1/2} b_{-1} c_{-1} \vert\tilde{\Omega}\rangle$	
	$\xi_0 c_0 \psi^\mu_{-1/2} b_{-1}\alpha^\nu_{-1} \vert\tilde{\Omega}\rangle$	

Table 2: Basis for Φ. Only those with ξ_0 and c_0 are shown. For the level-2 basis, the operator expressions are given only for those relevant to our calculation.

Level	Oscillator expression	Operator expression
0	$\xi_0\beta_{-1/2}\,|\widetilde{\Omega}\rangle$	$\xi\partial\xi c e^{-2\phi}e^{ik\cdot X}$
1	$\xi_0\beta_{-1/2}\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}\,|\widetilde{\Omega}\rangle$	$\xi\partial\xi c : \psi^{\mu}\psi^{\nu} : e^{-2\phi}e^{ik\cdot X}$
	$\xi_0\beta_{-1/2}\alpha^{\mu}_{-1}\,|\widetilde{\Omega}\rangle$	$\xi\partial\xi c\partial X^{\mu}e^{-2\phi}e^{ik\cdot X}$
	$\xi_0\psi^{\mu}_{-1/2}b_{-1}\,|\widetilde{\Omega}\rangle$	$\xi\psi^{\mu}e^{-\phi}e^{ik\cdot X}$
	$\xi_0\beta_{-3/2}\,|\widetilde{\Omega}\rangle$	$(\xi\partial^2\xi c e^{-2\phi} + \frac{1}{2}\xi\partial\xi c\partial e^{-2\phi})e^{ik\cdot X}$
	$\xi_0\beta_{-1/2}\beta_{-1/2}\gamma_{-1/2}\,|\widetilde{\Omega}\rangle$	$(\xi\partial\xi c\partial e^{-2\phi} + \xi\partial^2\xi c e^{-2\phi})e^{ik\cdot X}$
2	$\xi_0\beta_{-5/2}\,|\widetilde{\Omega}\rangle$	$(\xi\partial^3\xi e^{-2\phi} - 2\xi\partial^2\xi : \partial\phi e^{-2\phi} : +\xi\partial\xi : (\partial\phi\partial\phi - \partial^2\phi)e^{-2\phi} :)ce^{ik\cdot X}$
	$\xi_0\beta_{-3/2}\beta_{-1/2}\gamma_{-1/2}\,|\widetilde{\Omega}\rangle$	$(\xi\partial^2\xi\partial e^{-2\phi} + \frac{1}{2}\xi\partial^3\xi e^{-2\phi} + \frac{1}{2}\xi\partial\xi : (\partial\phi\partial\phi - \partial^2\phi)e^{-2\phi} :)ce^{ik\cdot X}$
	$\xi_0\beta_{-3/2}\alpha^{\mu}_{-1}\,|\widetilde{\Omega}\rangle$	$(\xi\partial^2\xi e^{-2\phi} + \frac{1}{2}\xi\partial\xi\partial e^{-2\phi})c : \partial X^{\mu}e^{ik\cdot X} :$
	$\xi_0\beta_{-3/2}\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}\,|\widetilde{\Omega}\rangle$	$(\xi\partial^2\xi e^{-2\phi} + \frac{1}{2}\xi\partial\xi\partial e^{-2\phi})c : \psi^{\mu}\psi^{\nu} : e^{ik\cdot X}$
	$\xi_0\beta_{-1/2}\beta_{-1/2}\beta_{-1/2}\gamma_{-1/2}\gamma_{-1/2}\,|\widetilde{\Omega}\rangle$	$(6\xi\partial\xi : \partial\phi\partial\phi e^{-2\phi} : +3\xi\partial^2\xi\partial e^{-2\phi} + \xi\partial^3\xi e^{-2\phi})ce^{ik\cdot X}$
	$\xi_0\beta_{-1/2}\beta_{-1/2}\gamma_{-3/2}\,|\widetilde{\Omega}\rangle$	$(\xi\partial\xi : (\partial\phi\partial\phi + \partial^2\phi)e^{-2\phi} : -\xi\partial^2\xi : \partial\phi e^{-2\phi} :)ce^{ik\cdot X}$
	$\xi_0\beta_{-1/2}\beta_{-1/2}\gamma_{-1/2}\alpha^{\mu}_{-1}\,|\widetilde{\Omega}\rangle$	$(\xi\partial\xi\partial e^{-2\phi} + \xi\partial^2\xi e^{-2\phi})c : \partial X^{\mu}e^{ik\cdot X} :$
	$\xi_0\beta_{-1/2}\beta_{-1/2}\gamma_{-1/2}\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}\,|\widetilde{\Omega}\rangle$	$(\xi\partial\xi\partial e^{-2\phi} + \xi\partial^2\xi e^{-2\phi})c : \psi^{\mu}\psi^{\nu} : e^{ik\cdot X}$
	$\xi_0\beta_{-1/2}\beta_{-1/2}c_{-1}\psi^{\mu}_{-1/2}\,|\widetilde{\Omega}\rangle$	$\xi\partial\xi\partial^2\xi c\partial^2 c e^{-3\phi}\psi^{\mu}e^{ik\cdot X}$
	$\xi_0\beta_{-1/2}\alpha^{\mu}_{-2}\,|\widetilde{\Omega}\rangle$	$\xi\partial\xi c e^{-2\phi} : \partial^2 X^{\mu}e^{ik\cdot X} :$
	$\xi_0\beta_{-1/2}\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\,|\widetilde{\Omega}\rangle$	$\xi\partial\xi c e^{-2\phi} : \partial X^{\mu}\partial X^{\nu}e^{ik\cdot X} :$
	$\xi_0\beta_{-1/2}\alpha^{\mu}_{-1}\psi^{\nu}_{-1/2}\psi^{\lambda}_{-1/2}\,|\widetilde{\Omega}\rangle$	$\xi\partial\xi c e^{-2\phi} : \psi^{\mu}\psi^{\nu} :: \partial X^{\lambda}e^{ik\cdot X} :$
	$\xi_0\beta_{-1/2}\psi^{\mu}_{-3/2}\psi^{\nu}_{-1/2}\,|\widetilde{\Omega}\rangle$	$\xi\partial\xi c e^{-2\phi} : \partial\psi^{\mu}\psi^{\nu} : e^{ik\cdot X}$
	$\xi_0\beta_{-1/2}\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}\psi^{\lambda}_{-1/2}\psi^{\rho}_{-1/2}\,|\widetilde{\Omega}\rangle$	$\xi\partial\xi c e^{-2\phi} : \psi^{\mu}\psi^{\nu}\psi^{\lambda}\psi^{\rho} : e^{ik\cdot X}$
	$\xi_0\beta_{-1/2}b_{-1}\gamma_{-1/2}\psi^{\mu}_{-1/2}\,|\widetilde{\Omega}\rangle$	$\xi\partial e^{-\phi}\psi^{\mu}e^{ik\cdot X}$
	$\xi_0\beta_{-1/2}b_{-1}c_{-1}\,|\widetilde{\Omega}\rangle$	$\xi\partial\xi\partial^2 c e^{-2\phi}e^{ik\cdot X}$
	$\xi_0 b_{-2}\psi^{\mu}_{-1/2}\,|\widetilde{\Omega}\rangle$	$\xi : bc : e^{-\phi}\psi^{\mu}e^{ik\cdot X}$
	$\xi_0 b_{-1}\psi^{\mu}_{-3/2}\,|\widetilde{\Omega}\rangle$	$\xi e^{-\phi}\partial\psi^{\mu}e^{ik\cdot X}$
	$\xi_0 b_{-1}\psi^{\mu}_{-1/2}\alpha^{\nu}_{-1}\,|\widetilde{\Omega}\rangle$	$\xi e^{-\phi}\psi^{\mu} : \partial X^{\nu}e^{ik\cdot X} :$
	$\xi_0 b_{-1}\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}\psi^{\lambda}_{-1/2}\,|\widetilde{\Omega}\rangle$	$\xi e^{-\phi} : \psi^{\mu}\psi^{\nu}\psi^{\lambda} : e^{ik\cdot X}$
	$b_{-2}b_{-1}\,|\Omega\rangle$	$be^{ik\cdot X}$

Table 3: Basis for Ω. Only those without c_0 are shown.

Level	Oscillator expression	Operator expression
0	none	
1	$\xi_0 c_0 \beta_{-1/2} b_{-1} \vert\widetilde{\Omega}\rangle$	$\xi\partial\xi\partial c e^{-2\phi} e^{ik\cdot X}$
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2}\psi^\mu_{-1/2} \vert\widetilde{\Omega}\rangle$	$\xi\partial\xi\partial^2\xi c\partial c\psi^\mu e^{-3\phi} e^{ik\cdot X}$
2	$\xi_0 c_0 \beta_{-3/2}\beta_{-1/2}\psi^\mu_{-1/2} \vert\widetilde{\Omega}\rangle$	$(\xi\partial\xi\partial^3\xi e^{-3\phi} - 2\xi\partial\xi\partial^2\xi : \partial\phi e^{-3\phi} :) c\partial c\psi^\mu e^{ik\cdot X}$
	$\xi_0 c_0 \beta_{-3/2} b_{-1} \vert\widetilde{\Omega}\rangle$	$(\xi\partial^2\xi e^{-2\phi} + \frac{1}{2}\xi\partial\xi\partial e^{-2\phi})\partial c e^{ik\cdot X}$
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2}\beta_{-1/2}\gamma_{-1/2}\psi^\mu_{-1/2} \vert\widetilde{\Omega}\rangle$	$(\xi\partial\xi\partial^2\xi\partial e^{-3\phi} + \xi\partial\xi\partial^3\xi e^{-3\phi}) c\partial c\psi^\mu e^{ik\cdot X}$
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2}\beta_{-1/2} c_{-1} \vert\widetilde{\Omega}\rangle$	$\xi\partial\xi\partial^2\xi\partial^3\xi e^{-4\phi} c\partial c\partial^2 c e^{ik\cdot X}$
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2}\psi^\mu_{-3/2} \vert\widetilde{\Omega}\rangle$	$\xi\partial\xi\partial^2\xi e^{-3\phi} c\partial c\partial\psi^\mu e^{ik\cdot X}$
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2}\psi^\mu_{-1/2}\psi^\nu_{-1/2}\psi^\lambda_{-1/2} \vert\widetilde{\Omega}\rangle$	$\xi\partial\xi\partial^2\xi e^{-3\phi} c\partial c : \psi^\mu\psi^\nu\psi^\lambda : e^{ik\cdot X}$
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2}\psi^\mu_{-1/2}\alpha^\nu_{-1} \vert\widetilde{\Omega}\rangle$	$\xi\partial\xi\partial^2\xi e^{-3\phi} c\partial c\psi^\mu : \partial X^\nu e^{ik\cdot X} :$
	$\xi_0 c_0 \beta_{-1/2}\beta_{-1/2} b_{-1}\gamma_{-1/2} \vert\widetilde{\Omega}\rangle$	$(\xi\partial\xi\partial e^{-2\phi} + \xi\partial^2\xi e^{-2\phi})\partial c e^{ik\cdot X}$
	$\xi_0 c_0 \beta_{-1/2} b_{-2} \vert\widetilde{\Omega}\rangle$	$\xi\partial\xi e^{-2\phi} : bc\partial c : e^{ik\cdot X}$
	$\xi_0 c_0 \beta_{-1/2} b_{-1}\alpha^\mu_{-1} \vert\widetilde{\Omega}\rangle$	$\xi\partial\xi e^{-2\phi}\partial c : \partial X^\mu e^{ik\cdot X} :$
	$\xi_0 c_0 \beta_{-1/2} b_{-1}\psi^\mu_{-1/2}\psi^\nu_{-1/2} \vert\widetilde{\Omega}\rangle$	$\xi\partial\xi e^{-2\phi}\partial c : \psi^\mu\psi^\nu : e^{ik\cdot X}$

Table 4: Basis for Ω. Only those with c_0 are shown.

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